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Effect of anisotropy and loss-cone on electrostatic drift instability in the presence of inhomogeneous magnetic field

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Abstract : The theory of particle aspect analysis has been developed to investigate the effect of the anisotropy and steepness of loss-cone on electrostatic drift instability in the presence of an inhomogeneous magnetic field. The whole plasma is considered to consist of resonant and non-resonant particles. Non-resonant particles support the oscillatory motion of the drift waves while the resonant particles participate in the energy exchange with the wave. A drift wave is assumed to start at $t = 0$ when the resonant particles are undisturbed. The trajectories of particles are then evaluated within the frame work of linear theory. Using these particles' trajectories in the presence of drift wave, the dispersion relation has been derived in electrostatic approximation and growth rate is evaluated by energy conservation method Effects of steepness of loss-cone distribution with temperature anisotropy are discussed on the dispersion relation and the growth rate of the instability. The results are derived for the space plasma parameters appropriate to the plasmapause region of the earth's magnetoplasma.

Keywords : Anisotropic and loss-cone, electrostatic drift instability

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1. Introduction

One likely region of space to exhibit the drift wave instability is the low energy thermal plasma at the plasmapause. The analysis of OGO-6 and OV 1-17 confirmed the existence of drift wave around $L = 4$ [1,2,3].

The study of drift plasma instabilities with finite β was conducted by several workers, where the effects of particle distribution anisotropy were also investigated [4–6]. Such a theory was initially developed with reference to the drift mirror instability [7].

The model under consideration introduces ∇B drifts in the orbits of the charged particles in the electrostatic approximation [4] and includes the resonant effects of the magnetic field gradient drifts for the electrons, considering the particle aspect analysis [8]. Charged particle trajectories are evaluated in the presence of an electrostatic drift wave propagating obliquely to the ambient magnetic field. The electromagnetic effects introduced by ∇ drifts have not been considered in the present analysis. Guiding centre theory is widely used in many problems of plasma physics. In order to obtain certain tokamak orbits, it may be necessary to extend this theory through second order by evaluating the trajectories in the presence of waves [9]. A low-β plasma is considered in the presence of an inhomogeneous magnetic field, which may be relevant to geomagnetic as well as laboratory plasmas.

The loss rate in the presence of loss-cone has been discussed in mirrors [10] and also in stellarators [11]. Recently, Itoh *et al* [12], derived an approximate form which describes the loss rate in stellarators under the influence of a loss-cone in the collisionless limit. Sharma [13] has investigated the gradient drift instability in the presence of an HF (high frequency) pump in the equatorial E-region including thermal non-linearity. In the auroral ionosphere, the interaction of HF pump wave with the gradient drift wave has been investigated in the presence of a steep loss-cone distribution function.

In this paper, particle aspect theories have been developed to investigate the effect of anisotropy and steepness of loss-cone on electrostatic drift instability in the presence of an inhomogeneous magnetic field in a low β plasma. These are based on Dawson's [14] theory of Landau damping, further extended by several workers [5,8,15–23] to the analysis of electrostatic and electromagnetic instabilities. The whole plasma is considered to consist of resonant and non-resonant particles. Non-resonant particles support the oscillatory motion of the drift waves while the resonant particles participate in the energy exchange with the wave. A drift wave is assumed to start at $t = 0$ when the resonant particles are undisturbed. The trajectories of particles are then evaluated within the frame work of linear theory. Using these particle trajectories in the presence of drift wave, the dispersion relation has been derived in electrostatic approximation and growth rate is evaluated by energy conservation method. Effects of steepness of loss-cone distribution with temperature anisotropy are discussed on the dispersion relation and the growth rate of the instability. The results are derived for space plasma parameters, appropriate to plasmapause region, but the results are relevant to laboratory plasma as well.

2. Basic assumptions and particle trajectories

The ambient magnetic field $\boldsymbol{B}_0$ is considered to be directed along the positive z-axis. The density varies along the negative x-axis, and the variation of the magnetic field is opposite to the density variation. In an anisotropic plasma, the wave is considered to propagate normal to the density gradient and in the (y, z) plane. The drift wave is assumed to start at $t = 0$, when the resonant particles are undisturbed. The particle trajectory is calculated in the frame work of linear theory. The main interest lies in the behaviour of those waves which satisfy the conditions $v_{T_i} \langle\langle \frac{\omega}{k_\parallel} \langle\langle v_{T_e}$, $\omega << \Omega_i$, Ω_e, $k_\perp^2 \rho_e^2 << k_\perp \rho_i^2 < 1$, where v_{T_i} and v_{T_e} are the thermal velocities of the ions and electrons along the magnetic field, $\Omega_{i,e}$ and $\rho_{i,e}$ are the gyrofrequencies and mean gyroradii of the two species, in the mean magnetic field. $k_\perp$ and $k_\parallel$

are the components of the wave vector k across and along the static magnetic field B and are taken to be positive. ω is the wave frequency.

Considering an electrostatic drift wave which satisfies the condition

$$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t = 0,$$

as the magnetic field associated with wave electric field has not been considered. Thus, the wave is assumed to be of the form

$$\phi = \phi_1 \cos\left(k_\perp y + k_\parallel z - \omega t\right) \tag{1}$$

with the electrostatic limit

$$\boldsymbol{E} = \boldsymbol{E}_\perp + \boldsymbol{E}_\parallel = -\nabla\phi, \tag{2}$$

where ϕ_1 is assumed to be slowly varying function of time t, and ϕ is the electric potential of the wave, the frequency ω is real and the density gradient is taken in the direction of the negative x-axis.

The equation of motion for the particle is

$$m \frac{d\boldsymbol{v}}{dt} = q\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}_0\right.$$

The Gaussian system of units are adopted in this treatment and interactions between particles are neglected. If $\boldsymbol{E}$ on the right hand side is considered to be a small perturbation, $\boldsymbol{v}$ can be expressed as a sum of the unperturbed velocity $\boldsymbol{V}$ and the perturbed velocity $\boldsymbol{u}$

$$\boldsymbol{v} = \boldsymbol{V} + \boldsymbol{u},$$

$\boldsymbol{u}$ is determined by the set of equations as given by Terashima [8]. Taking the trajectories of free gyration as Winske and Gary [4], the perturbed velocities can be evaluated as Tiwari *et al* [5].

3. Density variations

From conservation of particle number, the density perturbation n_1 is defined in a similar way as done by Terashima [8], that is

$$\frac{dn_1}{dt} = -(\nabla \boldsymbol{u})N - u_x \frac{dN}{dx}, \tag{3}$$

which can be derived from the equation of continuity. In the right hand side of eq. (3) all the quantities can be expressed as a function of variable t only. After integrating and transforming the variables, one can get the expression for the density perturbation $n_1(\boldsymbol{r}, t)$ for non-resonant and resonant particles as Tiwari *et al* [5].

To evaluate the dispersion relation and growth rate, the loss-cone distribution function has been taken from Misra and Tiwari [16],

$$N(V) = \frac{N_0 V_{\perp}^{2J}}{\pi^{3/2} V_{T_{\perp}}^{2(J+1)} V_{T_{\parallel}} J!} \exp\left[-\frac{V_{\perp}^2}{V_{T_{\perp}}^2} - \frac{V_{\parallel}}{V_{T_{\parallel}}^2}\right], \tag{4}$$

$$f_{\perp}(V_{\perp}) = \frac{V_{\perp}^{2J}}{\pi V_{T_{\perp}}^{2(J+1)} J!} \exp\left[-\frac{V_{\perp}^2}{V_{T_{\perp}}^2}\right], \tag{5}$$

$$f_{\parallel}(V_{\parallel}) = \frac{1}{\sqrt{\pi}} \frac{1}{V_{T_{\parallel}}} \exp\left[-\frac{V_{\parallel}^2}{V_{T_{\parallel}}^2}\right] \tag{6}$$

where $J = 0, 1, 2, ...$ is the distribution index and measures the steepness of the loss-cone feature [24],

$$V_{T_{\parallel}}^2 = \frac{2T_{\parallel}}{m}, \qquad V_{T_{\perp}}^2 = (J+1)^{-1}\left(\frac{2T_{\perp}}{m}\right).$$

For $J = 0$, $N(V)$ behaves as Maxwellian distribution function and for $J = \infty$, $N(V)$ behaves as Dirac Delta function. The distribution function (4) with different temperatures along and across the ambient magnetic field, defines the loss-cone distribution function and its steepness effects are given by the increasing values of distribution index J.

4. Dispersion relation

In the integrated perturbed density for non-resonant particles

$$\tilde{n}_{i,e} = \int_0^{\infty} 2\pi V_{\perp}\, dV_{\perp} \int_{-\infty}^{+\infty} dV_{\parallel}\, n_{i,e}\,(r,t), \tag{7}$$

the dispersion relation has been calculated [5] with the help of the previous equations followed by the use of the Poisson equation

$$\nabla \cdot E = k^2\phi = 4\pi e\left(\tilde{n}_i - \tilde{n}_e\right). \tag{8}$$

Providing the effect of the loss-cone ($J \neq 0$) in the treatment, the expression for the dispersion relation is approximated as

$$\omega \approx \frac{1}{2} \frac{T_{\parallel_e}}{T_{\perp_e}} k_{\perp} V_d^e \left\{1 - (J+1)\, b_i \left(1 + \frac{T_{\parallel_e}}{T_{\perp_i}}\right)\right\}$$

$$\times\left[1 + \left\{1 + \frac{4\omega_{Bi}}{k_{\perp} v_d^e} \frac{T_{\perp_e}}{T_{\parallel_e}} \left(1 + (J+1) b_i \frac{T_{\parallel_e}}{T_{\perp_i}}\right)\right\}^{1/2}\right], \tag{9}$$

where $b_i = \frac{1}{2} k_\perp^2 \rho_i^2$ and v_d^e is the pressure drift defined by

$$v_d^e = T_{\perp_e}/m_e \Omega_e \frac{1}{N} \frac{dN}{dx} .$$

$T_\perp$ and $T_\parallel$ are the perpendicular and parallel temperatures respectively, and $\omega_{B_{i,e}} = k_\perp v_B^{i,e}$ is the magnetic drift frequency, where $v_B^{i,e} = T_{\perp_{i,e}} \varepsilon_B / m_{i,e} \Omega_{i,e}$ and $\varepsilon_B = B^{-1} \frac{dB}{dx}$ is the inverse scale length of the magnetic field gradient, $m_{i,e}$ being the mass of the ion or electron. The dispersion relation given by eq. (9) coincides with that derived by Tiwari *et al* [5] when the effect of the loss-cone is zero (*i.e.*, $J = 0$).

The expansion of the resonant condition in powers of ε_B in the derivation of the dispersion relation eq. (9), removes the resonant particle effects of ∇B but permits to carry out the $v_\perp$ integration. Only zero-th order Bessel terms have been taken and J_{0s} terms have been expanded into a form which reduces the sixteen-fold sums to a mathematically soluble form. It should be noted that $v_d^e > 0$ for $dN/dx < 0$.

5. Energy balance and growth rate

The oscillatory motion of non-resonant electrons [8] contains the major part of the energy. The wave energy density per unit wave length W_ω is of the same order as that of the changes in the energy density of non-resonant electrons W_e *i.e.*

$$W_\omega \approx W_e = \frac{\lambda k_\parallel^2 \phi_1^2}{16\pi} \frac{\omega_{pe}}{k_\parallel^2 \left(T_{\parallel_e}/m_e\right)} . \tag{10}$$

The change of energy of the resonant electrons per unit wave length W_r is obtained as Tiwari *et al* [5] and the growth rate has been evaluated by taking the law of conservation of energy as

$$\frac{d}{dt}\left(W_\omega + W_r\right) = 0.$$

Finally, the real part of the growth rate is obtained as [5]

$$\frac{\gamma}{\Omega_i} = \pi^{1/2} \left(\frac{\omega}{k_\parallel V_{T_e}}\right) \exp\left[-\frac{\omega^2}{k_\parallel^2 V_{T_e}^2}\right] \left[\left(1 - 2\frac{\omega_{B_e}}{\omega} \frac{\omega^2}{k_\parallel^2 V_{T_e}^2}\right)\right.$$

$$\left. \times \left(\frac{k_\perp v_d^e}{\omega} - 1\right) + \frac{\omega_{B_e}}{\omega}\left(\frac{1}{2} \frac{k_\perp v_d^i}{\omega} - 1\right)\right] \frac{\omega}{\Omega_i} . \tag{11}$$

It should be noted that the expression for the growth rate is the same as Tiwari *et al* [5] with the only difference that in this expression [eq. (11)], the real frequency ω appears modified in accordance with the eq. (9). Thus, the distribution index modifies the dispersion relation and hence the growth rate is affected by the distribution index J. The resonant integral with magnetic field gradient has been evaluated in a number of ways [4,25]. In the present analysis, it is assumed that only electrons are in resonance with the wave and participate in energy-exchange with it [8]. The effect of ion Landau damping has not been taken into account. When $\omega_{B_e} = 0$, eq. (11) reduces to the growth rate of the drift waves with the usual condition $k_\perp v_d^e > \omega$ for the growth.

In the present paper, particle aspect analysis has been used to separate out contribution from resonant and non-resonant particles to derive the dispersion relation and growth rate for a universal drift instability. For $B \neq 0$, the well known stabilizing effect is obtained *via* numerical solution of the linearized Vlasov equation [4,26]. In this treatment, a low-β plasma has been considered in order to facilitate expansion of the resonant denominators and to allow the use of the electrostatic approximation. Higher β introduces electromagnetic effects, which have not been taken into account in the present analysis. Here, only the effect of a magnetic field gradient has been considered on the electrostatic dispersion relation and growth rate.

6. Results and discussion

Tiwari *et al* [5] have evaluated the dispersion relation and the growth rate for the drift wave in an inhomogeneous magnetized plasma with temperature anisotropy and inhomogeneity in magnetic field, when the magnetic field gradient is in the opposite direction to the density gradient. They have ignored the effect of the steepness of loss-cone on electrostatic drift instability in their studies. In the present analysis, the expression for the dispersion relation and growth rate have been derived including the effect of the steepness of loss-cone. The magnetic field gradient ∇B is related to density gradient ∇N by the relation $\varepsilon_B = -\frac{1}{2}\beta\varepsilon_N$. The effects of magnetic field gradients are equivalent to finite β $(= 2\mu_0 p/B_0^2)$ effects which have been examined in detail by several authors [4,26] employing different methods. The following parameters have been used to evaluate the dispersion relation and growth rate appropriate to the plasmapause region of the earth's magnetoplasma [27].

$$B_0 = 5 \times 10^{-7} \text{ Webers m}^{-2},$$

$$k_\parallel = 1.6 \times 10^{-7} \text{ m}^{-1},$$

$$v_d^e = -v_d^i = 2\text{m /s},$$

$$\rho_i = 170 \text{ m},$$

$$(1/N \; dN/dx)^{-1} = 6 \times 10^5 \text{m},$$

$$T_e = T_i = 1 \text{ eV}.$$

Figure 1 shows the variation of the real frequency ω with perpendicular wave number, $k_\perp(\mathrm{m}^{-1})$ for a low-β plasma ($\beta = 0.05$) having temperature ratio $T_{\parallel_e}/T_{\perp_e} = 1$ and different values of distribution index J. It is noticed that wave frequency ω decreases and frequency band reduces in width for higher values of J. Thus lower frequencies may be possible in narrow emission band of $k_\perp$, which may be due to the decrease of ion drift velocity by the averaging of wave field over the Larmor orbit in the presence of steep loss-cone distribution function.

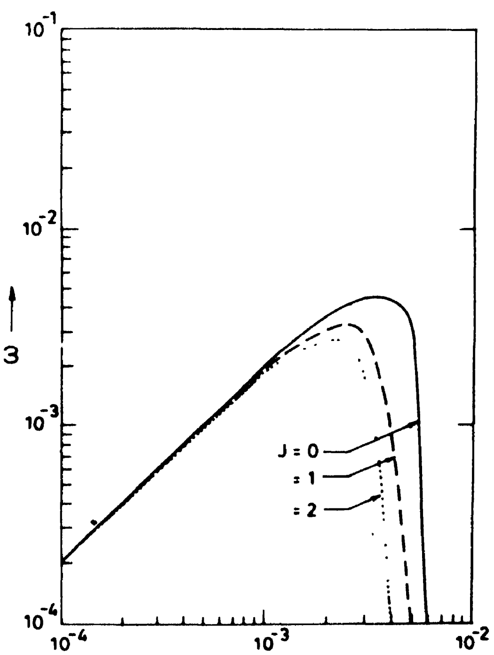

Figure 1. Variation of the real frequency ω with perpendicular wave number $k_\perp(\mathrm{m}^{-1})$ for different values of the distribution index J, having temperature anisotropy $T_{\parallel_e}/T_{\perp_e} = 1$ and $\beta = 0.05$.

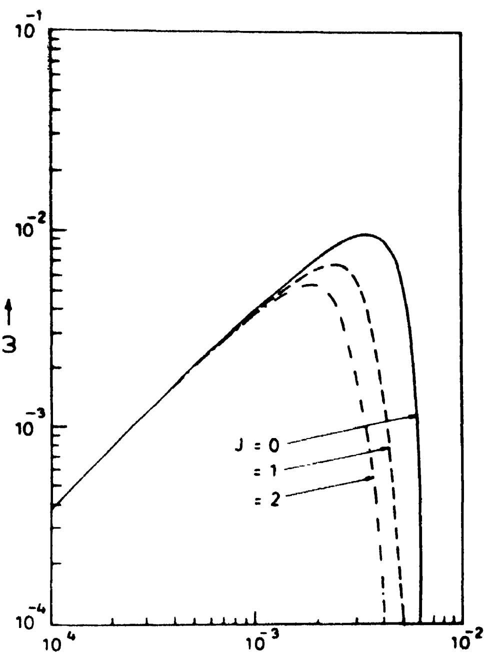

Figure 2. Variation of the real frequency ω with perpendicular wave number $k_\perp(\mathrm{m}^{-1})$ for different values of the distribution index J, having temperature anisotropy $T_{\parallel_e}/T_{\perp_e} = 2$ and $\beta = 0.05$.

Figure 2 shows variation of real frequency ω with perpendicular wave number $k_\perp(\mathrm{m}^{-1})$ for a low-β plasma ($\beta = 0.05$) having temperature ratio $T_{\parallel_e}/T_{\perp_e} = 2$ and different values of the distribution index J. In comparison to the previous case, it is noticed that wave frequency and the band-width increases considerably with the increase of the temperature ratio, although the tendency to decrease the peak value of wave frequency ω and contraction of the frequency band-width for higher values of J remains the same.

Figure 3 shows the variation of growth rate γ/Ω_i with $k_\perp$ for different values of J when $T_{\parallel_e}/T_{\perp_e} = 1$. The graph exhibits a gradual fall of the growth rate for the higher values of the loss-cone distribution index $J = 1$ and $J = 2$, within a certain range of $k_\perp$. The shifting of the maximum growth rate towards the lower values of $k_\perp$ is also clear in the figure. Thus, increase of J narrows the emission band and the wave can be excited for the lower $k_\perp$ values. Wave emissions of shorter wave number are possible for higher J and their growth is slower as compared to lower J.

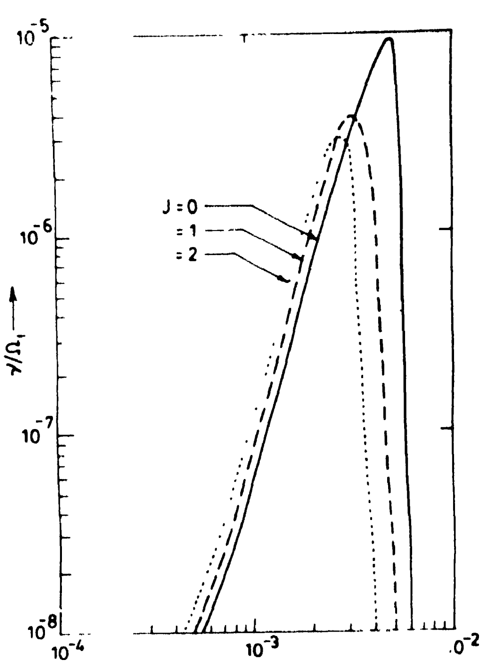

Figure 3. Variation of normalized growth rate γ/Ω_i with $k_\perp$ for different values of J, at $T_{\parallel_e}/T_{\perp_e} = 1$.

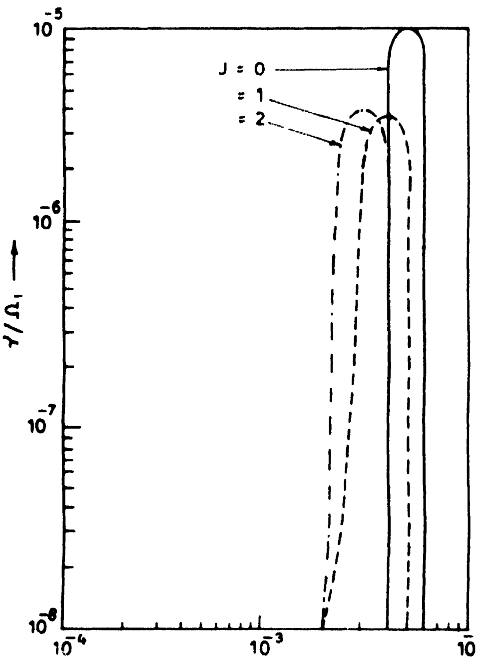

Figure 4. Variation of normalized growth rate γ/Ω_i with $k_\perp$ for different values of J, at $T_{\parallel_e}/T_{\perp_e} = 2$.

Figure 4 exhibits the variation of growth rate γ/Ω_i with $k_\perp$ for different values of J when $T_{\parallel_e}/T_{\perp_e} = 2$. The major contribution of the increase in the temperature ratio is to contract the emission bands for all the values of J. Although there is a fall in the peak value of the growth rate curve when the value of J is increased from zero to one, there is a slight increase in the peak value for $J = 2$.

Sharp density gradients appear in a variety of geophysical processes in the near space region of the earth, for example, the equatorial spread F and electrostatic emissions at the plasmapause. The equilibrium dipolar magnetic field of the earth is curved in the meridional plane and may introduce loss-cone effects in the particle distribution function. The index J measures the steepness of the loss-cone feature. Thus, the behaviour studied for the drift wave may be of importance in the electrostatic emissions around the plasmapause. The sharp density gradients may appear owing to the particle precipitation in the auroral zone [28].

Energetic particles may create the temperature anisotropy at the substorm times which may be the cause of drift wave emissions.

The theory may be useful for the hot electrons mirror experiments. The evolution of loss-cone in toroidal helical systems is inevitable in studying the potential applicability for the reactor [12,29]. The loss-cone problem would be serious for the alfa particles which are generated by nuclear fusion reaction in the toroidal helical systems. The study of loss-cone effects on the drift waves in such devices by evaluating the particles orbits may be useful for the loss and confinement of the thermonuclear plasmas. Single particle theory may be further extended to explain the energy exchange and heating of the plasma particles due to the drift wave. The currents associated with the wave may also be estimated by finding out the perturbed velocities and perturbed densities associated with the wave.

References

[1] P M Kintner and D A Gurnett *J. Geophys. Res.* **83** 39 (1978)

[2] M K Hudson and M C Kelley *J. Geophys Res.* **81** 3913 (1976)

[3] A F Nagy, L N Brace, N C Maynard and W B Hanson *J. Geophys Res.* **79** 435 (1974)

[4] D Winske and S P Gary *Phys Fluids* **22** 665 (1979)

[5] M S Tiwari, R P Pandey and K D Misra *J. Plasma Phys.* **34** 163 (1985)

[6] V A Marchenko, Yu M Nezlina and O A Pokhotelov *Plasma Phys Contr. Fusion* **30** 957 (1988)

[7] A Hasegawa *Phys Fluids* **12** 2642 (1969)

[8] Y Terashima *Prog. Theo. Phys* **37** 775 (1967)

[9] V P Milantiev and T Ortis *Plsama Phys* **24** 1493 (1982)

[10] V P Pastukhov *Nucl. Fusion* **14** 3 (1974)

[11] K Miyamoto *et al Phys. Fluids* **17** 1476 (1974)

[12] K Itoh, S I Itoh, A Gukuyama and K Hanatani *Nucl Fusion* **29** 1851 (1989)

[13] A K Sharma *J. Geophys Res.* **94** 3305 (1989)

[14] J M Dawson *Phys. Fluids* **4** 869 (1961)

[15] K D Misra and M S Tiwari *Physica Scripta* **16** 142 (1977)

[16] K D Misra and M S Tiwari *Can J Phys.* **57** 1124 (1979)

[17] K D Misra and M S Tiwari *Physica* **98C** 325 (1980)

[18] P Verma and M S Tiwari *Phys. Scripta* **44** 296 (1991)

[19] P Verma and M S Tiwari *Indian J. Pure Appl. Phys* **30** 1 (1992a)

[20] P Verma and M S Tiwari *Phys. Scripta* **45** 275 (1992b)

[21] P Verma and M S Tiwari *Indian J. Pure Appl. Phys.* (in press) (1994)

[22] M S Tiwari and P Verma *J. Plasma Phys.* **46** 49 (1993)

[23] M S Tiwari and P Verma *Indian J. Pure Appl Phys.* (in press) (1994)

[24] H K Wong, C S Wu and Jr J D Gaffey *Phys. Fluids* **28(9)** 2751 (1985)

[25] S P Gary and J J Sanderson *Phys. Fluids* **22** 1500 (1979)

[26] D E Hastings and J E Mc Cune *Phys. Fluids* **25** 509 (1982)

[27] A Hasegawa *J. Geophys. Res.* **76** 5361 (1971)

[28] M S Tiwari and G Rostoker *Planet. Space Sci.* **32** 1497 (1984)

[29] B A Carreras, N Daminguez, L Garcia, V E Lynch and J F Lyon *Nucl. Fusion* **28** 1195 (1988)